Essential Question How can you use exponents to write numbers?

The expression 3^5 is called a **power**. The **base** is 3. The **exponent** is 5.

Base $\rightarrow 3^5 \leftarrow$ Exponent

1 ACTIVITY: Using Exponent Notation

Work with a partner.

a. Complete the table.

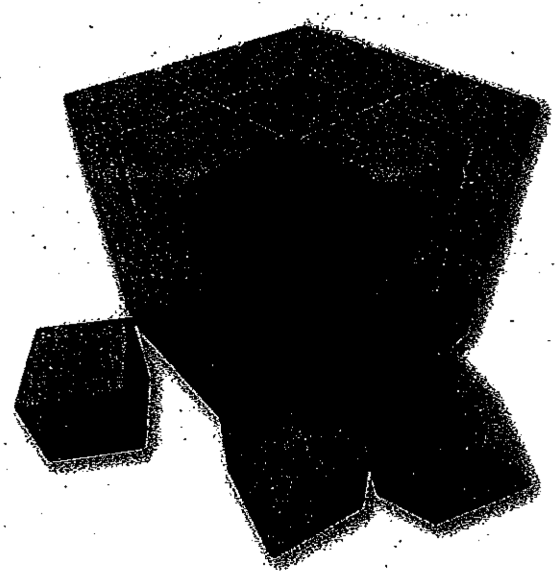
Power	Repeated Multiplication Form	Value
$(-3)^1$		
$(-3)^2$		
$(-3)^3$		
$(-3)^4$		
$(-3)^5$		
$(-3)^6$		
$(-3)^7$		

b. Describe what is meant by the expression $(-3)^n$. How can you find the value of $(-3)^n$?

2 ACTIVITY: Using Exponent Notation

Work with a partner.

a. The cube at the right has \$3 in each of its small cubes. Write a single power that represents the total amount of money in the large cube.



b. Evaluate the power to find the total amount of money in the large cube.

Lesson 1: Exponential Notation

Classwork

5^6 means $5 \times 5 \times 5 \times 5 \times 5 \times 5$ and $\left(\frac{9}{7}\right)^4$ means $\frac{9}{7} \times \frac{9}{7} \times \frac{9}{7} \times \frac{9}{7}$.

You have seen this kind of notation before, it is called **exponential notation**. In general, for any number x and any positive integer n,

$$x^n = \underbrace{(x \cdot x \cdots x)}_{n\ times}$$

The number x^n is called **x raised to the n-th power**, n is the **exponent** of x in x^n and x is the **base** of x^n.

Exercise 1

$\underbrace{4 \times \cdots \times 4}_{7\ times} =$

Exercise 2

$\underbrace{3.6 \times \cdots \times 3.6}_{\_\_\_\_\ times} = 3.6^{47}$

Exercise 3

$\underbrace{(-11.63) \times \cdots \times (-11.63)}_{34\ times} =$

Exercise 4

$\underbrace{12 \times \cdots \times 12}_{\_\_\_\_\ times} = 12^{15}$

Exercise 5

$\underbrace{(-5) \times \cdots \times (-5)}_{10\ times} =$

Exercise 6

$\underbrace{\frac{7}{2} \times \cdots \times \frac{7}{2}}_{21\ times} =$

Exercise 7

$\underbrace{(-13) \times \cdots \times (-13)}_{6\ times} =$

Exercise 8

$\underbrace{\left(-\frac{1}{14}\right) \times \cdots \times \left(-\frac{1}{14}\right)}_{10\ times} =$

Exercise 9

$\underbrace{x \cdot x \cdots x}_{185\ times} =$

Exercise 10

$\underbrace{x \cdot x \cdots x}_{\_\_\_\_\ times} = x^n$

Exercise 11

Will these products be positive or negative? How do you know?

$$\underbrace{(-1) \times (-1) \times \cdots \times (-1)}_{12\ times} = (-1)^{12}$$

$$\underbrace{(-1) \times (-1) \times \cdots \times (-1)}_{13\ times} = (-1)^{13}$$

Exercise 12

Is it necessary to do all of the calculations to determine the sign of the product? Why or why not?

$$\underbrace{(-5) \times (-5) \times \cdots \times (-5)}_{95\ times} = (-5)^{95}$$

$$\underbrace{(-1.8) \times (-1.8) \times \cdots \times (-1.8)}_{122\ times} = (-1.8)^{122}$$

Exercise 13

Fill in the blanks about whether the number is positive or negative.

If n is a positive even number, then $(-55)^n$ is ______________________.

If n is a positive odd number, then $(-72.4)^n$ is ______________________.

Exercise 14

Josie says that $\underbrace{(-15) \times \cdots \times (-15)}_{6\ times} = -15^6$. Is she correct? How do you know?

Problem Set

1. Use what you know about exponential notation to complete the expressions below.

$\underbrace{(-5) \times \cdots \times (-5)}_{17\ times} =$

$\underbrace{3.7 \times \cdots \times 3.7}_{\_\_\_\ times} = 3.7^{19}$

$\underbrace{7 \times \cdots \times 7}_{\_\_\_\ times} = 7^{45}$

$\underbrace{6 \times \cdots \times 6}_{4\ times} =$

$\underbrace{4.3 \times \cdots \times 4.3}_{13\ times} =$

$\underbrace{(-1.1) \times \cdots \times (-1.1)}_{9\ times} =$

$\underbrace{\left(\frac{2}{3}\right) \times \cdots \times \left(\frac{2}{3}\right)}_{19\ times} =$

$\underbrace{\left(-\frac{11}{5}\right) \times \cdots \times \left(-\frac{11}{5}\right)}_{\_\_\_\ times} = \left(-\frac{11}{5}\right)^{x}$

$\underbrace{(-12) \times \cdots \times (-12)}_{\_\_\_\ times} = (-12)^{15}$

$\underbrace{a \times \cdots \times a}_{m\ times} =$

2. Write an expression with (-1) as its base that will produce a positive product.

3. Write an expression with (-1) as its base that will produce a negative product.

4. Rewrite each number in exponential notation using 2 as the base.

$8 =$ $\qquad 16 =$ $\qquad 32 =$

$64 =$ $\qquad 128 =$ $\qquad 256 =$

5. Tim wrote 16 as $(-2)^4$. Is he correct?

6. Could -2 be used as a base to rewrite 32? 64? Why or why not?

Essential Question How can you multiply two powers that have the same base?

1 ACTIVITY: Finding Products of Powers

Work with a partner.

a. Complete the table.

Product	Repeated Multiplication Form	Power
$2^2 \bullet 2^4$		
$(-3)^2 \bullet (-3)^4$		
$7^3 \bullet 7^2$		
$5.1^1 \bullet 5.1^6$		
$(-4)^2 \bullet (-4)^2$		
$10^3 \bullet 10^5$		
$\left(\frac{1}{2}\right)^5 \bullet \left(\frac{1}{2}\right)^5$		

b. **INDUCTIVE REASONING** Describe the pattern in the table. Then write a rule for multiplying two powers that have the same base.

$a^m \bullet a^n = a^{\_\_\_}$

c. Use your rule to simplify the products in the first column of the table above. Does your rule give the results in the third column?

Essential Question How can you divide two powers that have the same base?

1 ACTIVITY: Finding Quotients of Powers

Work with a partner.

a. Complete the table.

Quotient	Repeated Multiplication Form	Power
$\frac{2^4}{2^2}$		
$\frac{(-4)^5}{(-4)^2}$		
$\frac{7^7}{7^3}$		
$\frac{8.5^9}{8.5^6}$		
$\frac{10^8}{10^5}$		
$\frac{3^{12}}{3^4}$		
$\frac{(-5)^7}{(-5)^5}$		
$\frac{11^4}{11^1}$		

b. INDUCTIVE REASONING Describe the pattern in the table. Then write a rule for dividing two powers that have the same base.

$$\frac{a^m}{a^n} = a^{\_\_\_}$$

Lesson 2: Multiplication of Numbers in Exponential Form

Classwork

In general, if x is any number and m, n are positive integers, then

$$x^m \cdot x^n = x^{m+n}$$

because

$$x^m \times x^n = \underbrace{(x \cdots x)}_{m\ times} \times \underbrace{(x \cdots x)}_{n\ times} = \underbrace{(x \cdots x)}_{m+n\ times} = x^{m+n}$$

Exercise 1

$14^{23} \times 14^8 =$

Exercise 2

$(-72)^{10} \times (-72)^{13} =$

Exercise 3

$5^{94} \times 5^{78} =$

Exercise 4

$(-3)^9 \times (-3)^5 =$

Exercise 5

Let a be a number.

$a^{23} \cdot a^8 =$

Exercise 6

Let f be a number.

$f^{10} \cdot f^{13} =$

Exercise 7

Let b be a number.

$b^{94} \cdot b^{78} =$

Exercise 8

Let x be a positive integer. If $(-3)^9 \times (-3)^x = (-3)^{14}$, what is x?

What would happen if there were more terms with the same base? Write an equivalent expression for each problem.

Exercise 9

$9^4 \times 9^6 \times 9^{13} =$

Exercise 10

$2^3 \times 2^5 \times 2^7 \times 2^9 =$

Can the following expressions be simplified? If so, write an equivalent expression. If not, explain why not.

Exercise 11

$6^5 \times 4^9 \times 4^3 \times 6^{14} =$

Exercise 12

$(-4)^2 \cdot 17^5 \cdot (-4)^3 \cdot 17^7 =$

Exercise 13

$15^2 \cdot 7^2 \cdot 15 \cdot 7^4 =$

Exercise 14

$2^4 \times 8^2 = 2^4 \times 2^6 =$

Exercise 15

$3^7 \times 9 =$

Exercise 16

$5^4 \times 2^{11} =$

Exercise 17

Let x be a number. Simplify the expression of the following number:

$(2x^3)(17x^7) =$

Exercise 18

Let a and b be numbers. Use the distributive law to simplify the expression of the following number:

$a(a + b) =$

Exercise 19

Let a and b be numbers. Use the distributive law to simplify the expression of the following number:

$b(a+b) =$

Exercise 20

Let a and b be numbers. Use the distributive law to simplify the expression of the following number:

$(a+b)(a+b) =$

In general, if x is nonzero and m, n are positive integers, then

$$\frac{x^m}{x^n} = x^{m-n} \qquad \text{if } m > n$$

Exercise 21

$\frac{7^9}{7^6} =$

Exercise 22

$\frac{(-5)^{16}}{(-5)^7} =$

Exercise 23

$\frac{\left(\frac{8}{5}\right)^9}{\left(\frac{8}{5}\right)^2} =$

Exercise 24

$\frac{13^5}{13^4} =$

Exercise 25

Let a, b be nonzero numbers. What is the following number?

$$\frac{\left(\frac{a}{b}\right)^9}{\left(\frac{a}{b}\right)^2} =$$

Exercise 26

Let x be a nonzero number. What is the following number?

$$\frac{x^5}{x^4} =$$

Can the following expressions be simplified? If yes, write an equivalent expression for each problem. If not, explain why not.

Exercise 27

$$\frac{2^7}{4^2} = \frac{2^7}{2^4} =$$

Lesson 29

$$\frac{3^5 \cdot 2^8}{3^2 \cdot 2^3} =$$

Exercise 28

$$\frac{3^{23}}{27} = \frac{3^{23}}{3^3} =$$

Lesson 30

$$\frac{(-2)^7 \cdot 95^5}{(-2)^5 \cdot 95^4} =$$

Lesson 3: Numbers in Exponential Form Raised to a Power

Classwork

For any number x and any positive integers m and n,

$$(x^m)^n = x^{mn}$$

because

$$\begin{aligned}(x^m)^n &= (\underbrace{x \cdot x \cdots x}_{m\ times})^n \\ &= \underbrace{(x \cdot x \cdots x)}_{m\ times} \times \cdots \times \underbrace{(x \cdot x \cdots x)}_{m\ times} \quad (n\ times) \\ &= x^{mn}\end{aligned}$$

Exercise 1

$(15^3)^9 =$

Exercise 2

$((-2)^5)^8 =$

Exercise 3

$(3.4^{17})^4 =$

Exercise 4

Let s be a number.

$(s^{17})^4 =$

Exercise 5

Sarah wrote that $(3^5)^7 = 3^{12}$. Correct her mistake. Write an exponential expression using a base of 3 and exponents of 5, 7, and 12 that would make her answer correct.

Exercise 6

A number y satisfies $y^{24} - 256 = 0$. What equation does the number $x = y^4$ satisfy?

For any numbers x and y, and positive integer n,

$$(xy)^n = x^n y^n$$

because

$$\begin{aligned}(xy)^n &= \underbrace{(xy)\cdots(xy)}_{n\ times} \\ &= \underbrace{(x\cdot x\cdots x)}_{n\ times}\cdot\underbrace{(y\cdot y\cdots y)}_{n\ times} \\ &= x^n y^n\end{aligned}$$

Exercise 7

$(11 \times 4)^9 =$

Exercise 8

$(3^2 \times 7^4)^5 =$

Exercise 9

Let a, b, and c be numbers.

$(3^2 a^4)^5 =$

Exercise 10

Let x be a number.

$(5x)^7 =$

Exercise 11

Let x and y be numbers.

$(5xy^2)^7 =$

Exercise 12

Let a, b, and c be numbers.

$(a^2bc^3)^4 =$

Exercise 13

Let x and y be numbers, $y \neq 0$, and let n be a positive integer. How is $\left(\frac{x}{y}\right)^n$ related to x^n and y^n?

Problem Set

1. Show (prove) in detail why $(2 \cdot 3 \cdot 4)^4 = 2^4 3^4 4^4$.

2. Show (prove) in detail why $(xyz)^4 = x^4 y^4 z^4$ for any numbers x, y, z.

3. Show (prove) in detail why $(xyz)^n = x^n y^n z^n$ for any numbers x, y, z, and for any positive integer n.

Lesson 4: Numbers Raised to the Zeroth Power

Classwork

For any numbers x, y, and any positive integers m, n, the following holds:

$$x^m \cdot x^n = x^{m+n} \quad (1)$$

$$(x^m)^n = x^{mn} \quad (2)$$

$$(xy)^n = x^n y^n \quad (3)$$

Definition: ______________________________

$n^0 =$

Here is the proof:

6. $10^3 \div 10^3 =$ ____________ = ____________ = ____________

7. $4^2 \div 4^2 =$ ____________ = ____________ = ____________

8. $3^1 \div 3^1 =$ ____________ = ____________ = ____________

Practice problems:

9. $3^3 \times 3^2 =$

12. $6^2 \times 6^1 =$

10. $4^5 \div 4^3 =$

13. $7^4 \times 7^2 =$

11. $10^3 \div 10^3 =$

14. $5^6 \div 5^4 =$

3 ACTIVITY: Writing a Definition

Work with a partner.

Compare the two methods used to simplify $\frac{3^2}{3^5}$. Then describe how you can rewrite a power with a negative exponent as a fraction.

Method 1

$$\frac{3^2}{3^5} = \frac{\cancel{3}^1 \bullet \cancel{3}^1}{{}_1\cancel{3} \bullet {}_1\cancel{3} \bullet 3 \bullet 3 \bullet 3}$$

$$= \frac{1}{3^3}$$

Method 2

$$\frac{3^2}{3^5} = 3^{2-5}$$

$$= 3^{-3}$$

Lesson 5: Negative Exponents and the Laws of Exponents

Classwork

Definition: For any positive number x and for any positive integer n, we define $x^{-n} = \frac{1}{x^n}$.

Note that this definition of negative exponents says x^{-1} is just the reciprocal $\frac{1}{x}$ of x.

As a consequence of the definition, for a positive x and all *integers b,* we get

$$x^{-b} = \frac{1}{x^b}$$

Exercise 1

Verify the general statement $x^{-b} = \frac{1}{x^b}$ for $x = 3$ and $b = -5$.

Exercise 2

What is the value of (3×10^{-2})?

Exercise 3

What is the value of (3×10^{-5})?

Exercise 4

Write the complete expanded form of the decimal 4.728 in exponential notation.

For Exercises 5–10, write an equivalent expression, in exponential notation, to the one given and simplify as much as possible.

Exercise 5

$5^{-3} =$

Exercise 6

$\frac{1}{8^9} =$

Exercise 7

$3 \cdot 2^{-4} =$

Exercise 8

Let x be a nonzero number.

$x^{-3} =$

Exercise 9

Let x be a nonzero number.

$\frac{1}{x^9} =$

Exercise 10

Let x, y be two nonzero numbers.

$xy^{-4} =$

We accept that for positive numbers x, y and all integers a and b,

$$x^a \cdot x^b = x^{a+b}$$

$$\left(x^b\right)^a = x^{ab}$$

$$(xy)^a = x^a y^a$$

We claim:

$$\frac{x^a}{x^b} = x^{a-b} \qquad \text{for all } integers\ a, b$$

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} \qquad \text{for any } integer\ a$$

Exercise 11

$\frac{19^2}{19^5} =$

Exercise 12

$\frac{17^{16}}{17^{-3}} =$

Exercise 13

If we let $b = -1$ in (11), a be any integer, and y be any positive number, what do we get?

Problem Set

1. Compute: $3^3 \times 3^2 \times 3^1 \times 3^0 \times 3^{-1} \times 3^{-2} =$
   Compute: $5^2 \times 5^{10} \times 5^8 \times 5^0 \times 5^{-10} \times 5^{-8} =$
   Compute. For a nonzero number, a: $a^m \times a^n \times a^l \times a^{-n} \times a^{-m} \times a^{-l} \times a^0 =$

2. Without using (10), show directly that $(17.6^{-1})^8 = 17.6^{-8}$.

3. Without using (10), show (prove) that for any whole number n and any positive number y, $(y^{-1})^n = y^{-n}$.

4. Show directly without using (13) that $\frac{2.8^{-5}}{2.8^7} = 2.8^{-12}$.