

LESSON 3.1 Representing Proportional Relationships

COMMON CORE 8.EE.6
 derive the equation $y = mx$ for a line through the origin. Also 8.F.4



ESSENTIAL QUESTION

How can you use tables, graphs, and equations to represent proportional situations?

EXPLORE ACTIVITY



COMMON CORE Prep for 8.EE.6

Representing Proportional Relationships with Tables

In 1870, the French writer Jules Verne published *20,000 Leagues Under the Sea*, one of the most popular science fiction novels ever written. One definition of a *league* is a unit of measure equaling 3 miles.

A Complete the table.

Distance (leagues)	1	2	6		20,000
Distance (miles)	3			36	

B What relationships do you see among the numbers in the table?

C For each column of the table, find the ratio of the distance in miles to the distance in leagues. Write each ratio in simplest form.

$$\frac{3}{1} = \square \quad \frac{\square}{2} = \square \quad \frac{\square}{6} = \square \quad \frac{36}{\square} = \square \quad \frac{\square}{20,000} = \square$$

D What do you notice about the ratios? _____

Reflect

- If you know the distance between two points in leagues, how can you find the distance in miles? _____
- If you know the distance between two points in miles, how can you find the distance in leagues? _____



Math On the Spot

© my.hrw.com

Representing Proportional Relationships with Equations

The ratio of the distance in miles to the distance in leagues is constant. This relationship is said to be *proportional*. A **proportional relationship** is a relationship between two quantities in which the ratio of one quantity to the other quantity is constant.

A proportional relationship can be described by an equation of the form $y = kx$, where k is a number called the **constant of proportionality**.

Sometimes it is useful to use another form of the equation, $k = \frac{y}{x}$.

EXAMPLE 1



COMMON CORE 8.EE.6

Meghan earns \$12 an hour at her part-time job. Show that the relationship between the amount she earned and the number of hours she worked is a proportional relationship. Then write an equation for the relationship.

STEP 1 Make a table relating amount earned to number of hours.

For every hour Meghan works, she earns \$12. So, for 8 hours of work, she earns $8 \times \$12 = \96 .

Number of hours	1	2	4	8
Amount earned (\$)	12	24	48	96

STEP 2 For each number of hours, write the relationship of the amount earned and the number of hours as a ratio in simplest form.

$$\frac{\text{amount earned}}{\text{number of hours}} \quad \frac{12}{1} = \frac{12}{1} \quad \frac{24}{2} = \frac{12}{1} \quad \frac{48}{4} = \frac{12}{1} \quad \frac{96}{8} = \frac{12}{1}$$

Since the ratios for the two quantities are all equal to $\frac{12}{1}$, the relationship is proportional.

STEP 3 Write an equation.

First tell what the variables represent.

Let x represent the number of hours.
Let y represent the amount earned.

Use the ratio as the constant of proportionality in the equation $y = kx$.

The equation is $y = \frac{12}{1}x$ or $y = 12x$.

Math Talk

Mathematical Practices

Describe two real-world quantities with a proportional relationship that can be described by the equation $y = 25x$.

YOUR TURN

3. Fifteen bicycles are produced each hour at the Speedy Bike Works. Show that the relationship between the number of bikes produced and the number of hours is a proportional relationship. Then write an equation for the relationship.



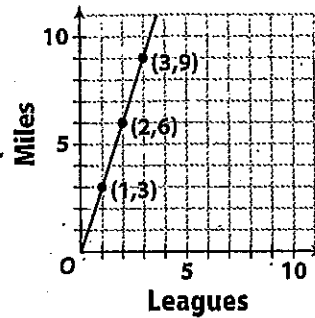
Personal Math Trainer

Online Assessment and Intervention

© my.hrw.com

Representing Proportional Relationships with Graphs

You can represent a proportional relationship with a graph. The graph will be a line that passes through the origin (0, 0). The graph shows the relationship between distance measured in miles to distance measured in leagues.

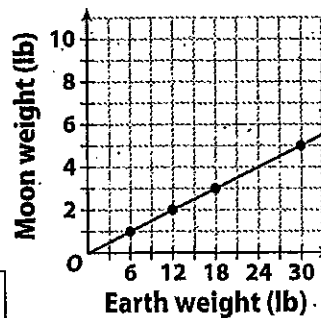


EXAMPLE 2



COMMON CORE 8.EE.6

The graph shows the relationship between the weight of an object on the Moon and its weight on Earth. Write an equation for this relationship.



STEP 1 Use the points on the graph to make a table.

Earth weight (lb)	6	12	18	30
Moon weight (lb)	1	2	3	5

STEP 2 Find the constant of proportionality.

$$\frac{\text{Moon weight}}{\text{Earth weight}} = \frac{1}{6} = \frac{1}{6} \quad \frac{2}{12} = \frac{1}{6} \quad \frac{3}{18} = \frac{1}{6} \quad \frac{5}{30} = \frac{1}{6}$$

The constant of proportionality is $\frac{1}{6}$.

STEP 3 Write an equation.

Let x represent weight on Earth.

Let y represent weight on the Moon.

The equation is $y = \frac{1}{6}x$. Replace k with $\frac{1}{6}$ in $y = kx$.

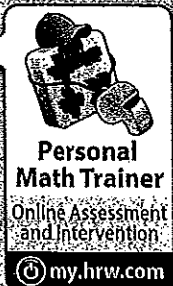
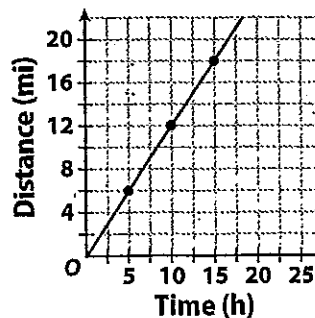


YOUR TURN

The graph shows the relationship between the amount of time that a backpacker hikes and the distance traveled.

- What does the point (5, 6) represent?

- What is the equation of the relationship?



© Houghton Mifflin Harcourt Publishing Company • Image Credits: © David Epperson/PhotoDisc/Getty Images

Guided Practice

1. **Vocabulary** A proportional relationship is a relationship between two quantities in which the ratio of one quantity to the other quantity

is / is not constant.

2. **Vocabulary** When writing an equation of a proportional relationship in the form $y = kx$, k represents the _____.

3. Write an equation that describes the proportional relationship between the number of days and the number of weeks in a given length of time. (Explore Activity and Example 1)

- a. Complete the table.

Time (weeks)	1	2	4		10
Time (days)	7			56	

- b. Let x represent _____.

Let y represent _____.

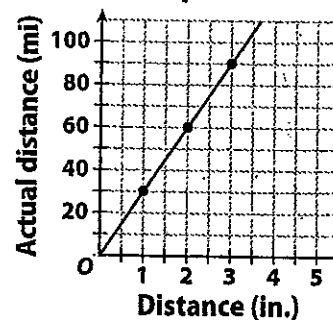
The equation that describes the relationship is _____.

Each table or graph represents a proportional relationship. Write an equation that describes the relationship. (Example 1 and Example 2)

4. **Physical Science** The relationship between the numbers of oxygen atoms and hydrogen atoms in water

Oxygen atoms	2	5		120
Hydrogen atoms	4		34	

5. **Map of Iowa**



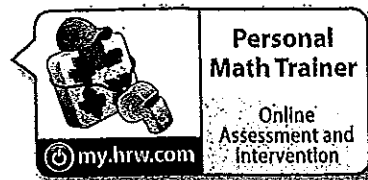
ESSENTIAL QUESTION CHECK-IN

6. If you know the equation of a proportional relationship, how can you draw the graph of the equation?

3.1 Independent Practice

COMMON
CORE

8.EE.6, 8.F.4



The table shows the relationship between temperatures measured on the Celsius and Fahrenheit scales.

Celsius temperature	0	10	20	30	40	50
Fahrenheit temperature	32	50	68	86	104	122

7. Is the relationship between the temperature scales proportional? Why or why not?

8. Describe the graph of the Celsius-Fahrenheit relationship.

9. **Analyze Relationships** Ralph opened a savings account with a deposit of \$100. Every month after that, he deposited \$20 more.

- a. Why is the relationship described not proportional?

- b. How could the situation be changed to make the situation proportional?

10. **Represent Real-World Problems** Describe a real-world situation that can be modeled by the equation $y = \frac{1}{20}x$. Be sure to describe what each variable represents.

Look for a Pattern The variables x and y are related proportionally.

11. When $x = 8$, $y = 20$. Find y when $x = 42$. _____
12. When $x = 12$, $y = 8$. Find x when $y = 12$. _____

13. The graph shows the relationship between the distance that a snail crawls and the time that it crawls.

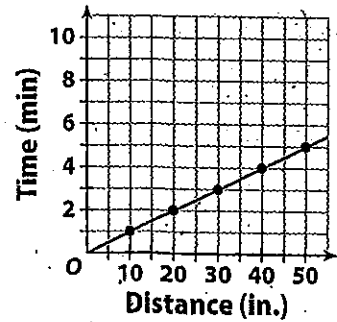
a. Use the points on the graph to make a table.

Distance (in.)					
Time (min)					

b. Write the equation for the relationship and tell what each variable represents.

c. How long does it take the snail to crawl 85 inches? _____

Snail Crawling



H.O.T. FOCUS ON HIGHER ORDER THINKING

14. **Communicate Mathematical Ideas** Explain why all of the graphs in this lesson show the first quadrant but omit the other three quadrants.

15. **Analyze Relationships** Complete the table.

Length of side of square	1	2	3	4	5
Perimeter of square					
Area of square					

a. Are the length of a side of a square and the perimeter of the square related proportionally? Why or why not?

b. Are the length of a side of a square and the area of the square related proportionally? Why or why not?

16. **Make a Conjecture** A table shows a proportional relationship where k is the constant of proportionality. The rows are then switched. How does the new constant of proportionality relate to the original one?

Work Area